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THE EFFECTS OF YAWING THIN POINTED

WINGS AT SUPERSONIC SPEEDS

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SUMMARY

An approximate relation is derived for the surface velocity
potential of thin pointed wings at supersonic speeds when they are
contained within the Mach cone from the vertex. This relation is
applied to obtain the pressure distributions, the lift and drag
coefficients, the center of pressure, and the rolling moments as
a function of angle of yaw for the delta wing. Theoretical curves
are presented for a Mach number of $\sqrt{2}$ to illustrate the relations.

INTRODUCTION

The linearized equation of compressible flow at supersonic
speeds has been applied by Stewart (reference 1) and Brown (refer-
ence 2) to obtain the lift distribution of a thin delta wing with-
out yaw. Special cases of yawed delta wings have been solved by
Hayes of North American Aviation, Inc. All these solutions are
simplified by the fact that the flow is conical. This simplifica-
tion is inapplicable if the leading edges are curved.

A general method has been developed (the basic principle of
which is presented in reference 3) for obtaining the lift distri-
bution of thin wings of arbitrary plan form and profile. Although
the method may be applied in principle to obtain the effects of
yawing the thin pointed wing, the practical evaluation of the
integrals appears to be difficult even by numerical methods. Never-
theless, an approximate solution to the problem may be obtained
that will indicate the effects of the plan-form leading-edge curva-
ture and of yaw on the aerodynamic coefficients of the wing.

An approximate solution for the surface velocity potential of
thin pointed wings at supersonic speeds when the leading edges are
included within the Mach cone from the vertex was developed during
March 1947 at the NACA Cleveland laboratory and is presented
herein. The solution is applied to calculate the pressure

distribution, the center of pressure, and the wave lift and drag coefficients of the delta wing as functions of yaw angle.

ANALYSIS

The basis of the derivation for the surface velocity potential is to replace the influence of the pointed wing and the flow field between the leading edge and the foremost Mach waves by an approximately equivalent wing surface without external disturbing flow fields when the velocity potential at some local point on the wing is calculated. The approximations that are applied will be described in detail for the case of the delta wing.

When the leading edges of the delta wing are swept back along the Mach lines (fig. 1(a)), there is no external flow field representing interaction between the top and bottom wing surfaces. The correct values of the velocity potential at any point on the top wing surface may therefore be obtained through the methods of reference 4 by an integration over the parallelogram area S_w bounded by the leading edge and the forward Mach cone from the point (x,y); that is

$$\varphi = \frac{U\alpha}{\pi} \iint_{S_w} \frac{d\xi\, d\eta}{\sqrt{(x-\xi)^2 - \beta^2(y-\eta)^2}} \tag{1}$$

where

φ velocity potential at point (x,y) on top wing surface

U free-stream velocity

α angle of attack (negative flow-deflection angle on top wing surface in $y = $ constant plane)

x or ξ, y or η Cartesian coordinates

β $\sqrt{M^2-1}$

M free-stream Mach number

(See references 3 or 4.)

If the wing is swept behind the Mach cone from the vertex but is so yawed that one of the leading edges coincides with the Mach line (fig. 1(b)), the velocity potential will still be correctly obtained by integrating equation (1) over the parallelogram area $S_{w,0}$ defined by the forward Mach lines from the point (x,y) and the wing leading edges.

According to the methods of reference 3, the contribution to the potential at point (x,y) from the external flow field S_D is

$$\varphi_D = -\alpha \iint_{S_{w,1}} \frac{d\xi\ d\eta}{\sqrt{(x-\xi)^2 - \beta^2(y-\eta^2)}} = -\varphi_{w,1}$$

where $\varphi_{w,1}$ is the portion of the potential contributed by the wing region $S_{w,1}$. The external field S_D thus effectively cancels the influence of the wing portion $S_{w,1}$.

The case including yaw with both leading edges swept behind the Mach cone from the vertex may now be considered. (See fig. 1(c).) The potential at point (x,y) is influenced by both the wing surfaces $S_{w(0+1+2+3)}$ and the external fields off the wing surfaces $S_{D(1+2+3+4)}$. The potential is then

$$\varphi = \frac{U\alpha}{\pi} \iint_{S_{w(0+1+2+3)}} \frac{d\xi\ d\eta}{\sqrt{(x-\xi)^2 - \beta^2(y-\eta)^2}}$$

$$- \frac{U}{\pi} \iint_{S_{D(1+2+3+4)}} \frac{\lambda d\xi\ d\eta}{\sqrt{(x-\xi)^2 - \beta^2(y-\eta)^2}} \tag{2}$$

where λ represents the slope of the stream sheet in the external flow field near the x,y plane measured in $\eta = $ constant planes. By the methods of reference 3

$$\iint_{S_{D(2+3+4)}} \frac{\lambda d\xi\, d\eta}{\sqrt{(x-\xi)^2 - \beta^2(y-\eta)^2}} = \iint_{S_{W(2+3)}} \frac{\alpha\, d\xi\, d\eta}{\sqrt{(x-\xi)^2 - \beta^2(y-\eta)^2}} \qquad (3)$$

and

$$\iint_{S_{D(1+3+4)}} \frac{\lambda d\xi\, d\eta}{\sqrt{(x-\xi)^2 - \beta^2(y-\eta)^2}} = \iint_{S_{W(1+3)}} \frac{\alpha d\xi\, d\eta}{\sqrt{(x-\xi)^2 - \beta^2(y-\eta)^2}} \qquad (4)$$

Substitution of equations (3) and (4) into equation (2) yields

$$\varphi = \frac{U\alpha}{\pi} \iint_{S_{W,0}} \frac{d\xi\, d\eta}{\sqrt{(x-\xi)^2 - \beta^2(y-\eta)^2}} - \frac{U\alpha}{\pi} \iint_{S_{W,3}} \frac{d\xi\, d\eta}{\sqrt{(x-\xi)^2 - \beta^2(y-\eta)^2}} + \frac{U}{\pi} \iint_{S_{D(3+4)}} \frac{\lambda d\xi\, d\eta}{\sqrt{(x-\xi)^2 - \beta^2(y-\eta)^2}}$$

$$(5)$$

Because λ and α have the same sign (reference 3), the second and third integrals of equation (5) tend to counteract each other. Furthermore, as the sweepback of the leading edges approaches the Mach lines from the vertex, the areas $S_{W,3}$ and $S_{D(3+4)}$ approach zero and $S_{W,0}$ increases. (The limiting case is shown in fig. 1(a).) The second and third integrals of equation (5) may therefore be neglected, provided that the sweepback is not too much greater than the Mach angle. Within the validity of this approximation, the potential may be calculated from the shaded area $S_{W,0}$ (fig. 1(c)).

A comparison of the results thus far obtained shows that the parallelogram representation of the thin delta wing is exact for the two limiting cases (fig. 1(a) and fig. 1(b)), but is approximate for the general case (fig. 1(c)). The error in the approximation is likely to be greatest for wings swept considerably behind the Mach cone from the vertex and for wings at zero angle of yaw. The parallelogram approximation may likewise be applied when the leading edges of the wing are gently curved on the plan form, as in figure 2.

A set of oblique coordinates (u,v) having axes parallel to the Mach waves simplifies the calculation of the velocity potential. In this sytem one of the coordinates of a point is the distance measured parallel to the coordinate axis from the point to the other coordinate axis. The transformation equations from Cartesian to oblique coordinates are

$$u = \frac{M}{2\beta} (\xi - \beta\eta)$$

$$v = \frac{M}{2\beta} (\xi + \beta\eta)$$

$$\xi = \frac{\beta}{M} (v+u)$$

$$\eta = \frac{1}{M} (v-u) \tag{6}$$

Inasmuch as the elemental area in the (u,v) coordinate system is $\frac{2\beta}{M^2}$ du dv, transformation of equation (1) by relations (6) yields

$$\varphi = \frac{U\alpha}{\pi M} \iint_{S_{w,0}} \frac{du\ dv}{\sqrt{(u_w-u)(v_w-v)}} \tag{7}$$

where u_w and v_w are the oblique coordinates of the point (x,y). Equation (7) becomes (see fig. 2)

$$\varphi = \frac{U\alpha}{\pi M} \int_{u_2(v_w)}^{u_w} \frac{du}{\sqrt{u_w - u}} \int_{v_1(u_w)}^{v_w} \frac{dv}{\sqrt{v_w - v}}$$

$$= \frac{4U\alpha}{\pi M} \sqrt{\left[v_w - v_1(u_w)\right] \left[u_w - u_2(v_w)\right]} \tag{8}$$

In the special case of the delta wing (fig. 3), the equations for the leading edges are

$$v = v_1(u) = k_1 u$$

and

$$u = u_2(v) = k_2 v$$

where k_1 and k_2 are positive constants.

In this case, equation (8) becomes

$$\varphi = \frac{4U\alpha}{\pi M} \sqrt{(v_w - k_1 u_w)(u_w - k_2 v_w)}$$

$$= \frac{2U\alpha}{\pi \beta} \sqrt{\left[x(1-k_1) + \beta y(1+k_1)\right]\left[x(1-k_2) - \beta y(1+k_2)\right]} \tag{9}$$

Now the pressure coefficient C_p is

$$C_p = \frac{\Delta p}{q} = -\frac{2}{U}\frac{\partial \varphi}{\partial x} \tag{10}$$

where Δp is the local static pressure minus the free-stream static pressure and q is the incompressible dynamic pressure $\frac{1}{2}\rho U^2$ (ρ is density). Insertion of equation (9) into equation (10) yields

$$C_p = -\frac{2\alpha}{\pi \beta} \sqrt{(1-k_1)(1-k_2)} \left[\sqrt{\frac{1 + \frac{1+k_1}{1-k_1}\frac{\beta y}{x}}{1 - \frac{1+k_2}{1-k_2}\frac{\beta y}{x}}} + \sqrt{\frac{1 - \frac{1+k_2}{1-k_2}\frac{\beta y}{x}}{1 + \frac{1+k_1}{1-k_1}\frac{\beta y}{x}}}\right] \tag{11}$$

By application of equations (6) and with the aid of figure 3,

$$k_1 = \frac{1 - \beta \tan(\theta+\psi)}{1 + \beta \tan(\theta+\psi)}$$

$$k_2 = \frac{1 - \beta \tan(\theta-\psi)}{1 + \beta \tan(\theta-\psi)} \tag{12}$$

where ψ is the angle of yaw and 2θ is the vertex angle of the delta wing. Substitution of equation (12) into equation (11) yields

$$C_p = -\frac{4\alpha}{\pi} \sqrt{\frac{\tan(\theta+\psi)\,\tan(\theta-\psi)}{[1+\beta\,\tan(\theta+\psi)]\,[1+\beta\,\tan(\theta-\psi)]}} \left[\sqrt{\frac{1 + \frac{y/x}{\tan(\theta+\psi)}}{1 - \frac{y/x}{\tan(\theta-\psi)}}} \right.$$

$$\left. + \sqrt{\frac{1 - \frac{y/x}{\tan(\theta-\psi)}}{1 + \frac{y/x}{\tan(\theta+\psi)}}} \right] \tag{13}$$

This equation indicates the pressure distribution on the thin delta wing swept behind the Mach angle at angles of yaw. The equation is exact if $\tan\theta$ or $\tan(\theta+\psi)$ or $\tan(\theta-\psi)$ is equal to $1/\beta$. The greatest error should occur when $\psi = 0$ and $\tan\theta \ll 1/\beta$. The accuracy of the expression increases with angle of yaw as long as the wing lies within the Mach cone from the vertex.

The case of $\psi = 0$ has been solved exactly by the authors of references 1 and 2. For $\psi = 0$, the approximate equation (13) reduces to

$$C_p = \frac{4}{\pi(1+\beta\,\tan\theta)} \left[\frac{-2\alpha\,\tan\theta}{\sqrt{1 - \frac{(y/x)^2}{\tan^2\theta}}} \right] \tag{14}$$

The bracketed portion of equation (14) gives the same type variation of the pressure coefficient over the wing surface as was obtained in references 1 and 2. The factor $\frac{4}{\pi(1+\beta\,\tan\theta)}$, however, is somewhat different. The uncorrected values of the factor

are compared with the true values of the factor (obtained from the expressions of reference 1 and the table of complete elliptic

integrals E of the second kind with modulus $\sqrt{1-\beta^2\tan^2\theta}$, reference 5) in the following table:

$\beta\tan\theta$	Uncorrected value of factor, F' $\dfrac{4}{\pi(1+\beta\tan\theta)}$	True value of factor, F $\dfrac{1}{E(\sqrt{1-\beta^2\tan^2\theta})}$
1.00	0.6366	0.6366
.9	.6701	.6697
.8	.7074	.7052
.7	.7490	.7432
.6	.7958	.7835
.5	.8488	.8257
.4	.9095	.8690
.3	.9794	.9121
.2	1.0610	.9520
.1	1.1575	.9842
0	1.2732	1.0000

Equation (14) (and, consequently, equation (13)) thus appears reasonably accurate, even when $\beta\tan\theta$ is as low as 0.5.

In numerical calculations involving equation (13), the accuracy of the factor

$$F' = \frac{4}{\pi\sqrt{[1+\beta\tan(\theta+\psi)][1+\beta\tan(\theta-\psi)]}}$$ (15)

may be improved by correction to give the true value at $\psi = 0$. The original F' and the estimated value of F are plotted against ψ and θ for $\beta = 1$ (that is, $M = \sqrt{2}$) in figure 4. In practice, for small angles of yaw, F may be set equal to the correct value at $\psi = 0$ of

$$F = \frac{1}{E(\sqrt{1-\beta^2\tan^2\theta})} \qquad \psi \approx 0$$

(The exact value of the factor F has recently been derived by Max A. Heaslet, Harvard Lomax, and Arthur L. Jones of the NACA Ames laboratory and is, in the notation of this paper,

$$F = \frac{1}{E} \sqrt{\frac{2G}{\beta \left[\tan(\theta+\psi) + \tan(\theta-\psi)\right]}}$$

where E is the complete elliptic integral of the second kind with modulus equal to $\sqrt{1-G^2}$ and

$$G = \frac{1+\beta^2 \tan(\theta+\psi) \tan(\theta-\psi) - \sqrt{\left[1-\beta^2 \tan^2(\theta+\psi)\right]\left[1-\beta^2 \tan^2(\theta-\psi)\right]}}{\beta \left[\tan(\theta+\psi) + \tan(\theta-\psi)\right]}$$

For pressure-distribution studies, the pressure coefficient is conveniently defined with respect to a set of coordinates fixed on the wing. The pressure coefficient in terms of the wing coordinate system (x_1,y_1) is derived in the appendix as equation (A3) and is

$$C_p = -\alpha F \sqrt{\frac{\tan^2\theta - \tan^2\psi}{1-\tan^2\theta \tan^2\psi}} \left[\sqrt{\frac{\left(\frac{\tan\theta}{\tan\theta+\tan\psi}\right) + \left(\frac{1}{\tan\theta+\tan\psi}\right)\frac{y_1}{x_1}}{\left(\frac{\tan\theta}{\tan\theta-\tan\psi}\right) - \left(\frac{1}{\tan\theta-\tan\psi}\right)\frac{y_1}{x_1}}} \right.$$

$$\left. + \sqrt{\frac{\left(\frac{\tan\theta}{\tan\theta-\tan\psi}\right) - \left(\frac{1}{\tan\theta-\tan\psi}\right)\frac{y_1}{x_1}}{\left(\frac{\tan\theta}{\tan\theta+\tan\psi}\right) + \left(\frac{1}{\tan\theta+\tan\psi}\right)\frac{y_1}{x_1}}} \right] \qquad (16)$$

The wave lift coefficient C_L of the yawed wing may be determined by integrating the pressure coefficient over the wing surface. This integration is performed in the appendix as equation (A5) to give

$$C_L = \frac{2\pi\alpha F \tan\theta}{\sqrt{1 - \tan^2\theta \tan^2\psi}} \qquad \left|\theta\pm\psi\right| \leq \tan^{-1}\frac{1}{\beta} \qquad (17)$$

A comparison of equations (16) and (17) shows that the lift coefficient is related to the pressure coefficient at the center of the wing through the equation

$$C_L = -\pi \, C_p \tag{18}$$

where C_p is evaluated at $y_1 = 0$. This relation does not contain the coefficient F and hence is exact. Variations of Mach number, angles of attack and yaw, and vertex half-angle of the delta wing swept behind the Mach angle do not alter the relation. A simple experimental pressure measurement will thus give the lift coefficient of the delta wing. By the principles of superposition, the pressure coefficient of the thin delta wing may be obtained as one-half the difference of the pressure coefficients on the top and bottom surfaces of a symmetrical-profile finite-thickness delta wing of the same plan form. The wave drag coefficient (neglecting leading-edge suction) is α times the lift coefficient or, with the use of equation (17), is

$$C_D = \frac{2\pi\alpha^2 \, F \, \tan\theta}{\sqrt{1 - \tan^2\theta \, \tan^2\psi}} \tag{19}$$

Thus both the wave lift and the wave drag coefficients increase with yaw (primarily because of the variation in F, fig. 3), as does $\dfrac{\partial C_L}{\partial \alpha}$. It should be noted, however, that α is the flow-deflection angle of attack of the yawed wing; the relation between α and the angle α', which the delta wing (rotated about its base) makes with the plane of zero angle of attack, is

$$\alpha = \alpha' \cos\psi \tag{20}$$

Inasmuch as the flow is conical on the yawed delta wing, the center of pressure lies along a line parallel to the base at a distance $\frac{2}{3} x_c$ (where x_c is the maximum chord) from the vertex. As shown in the appendix (equation (A10)), the wing coordinates of the center of pressure are given as

$$\left.\begin{array}{l} \overline{x_1} = \dfrac{2}{3} \, x_c \\[2mm] \overline{y_1} = -\dfrac{x_c}{3} \, \tan\psi \end{array}\right\} \tag{21}$$

The center of pressure therefore lies on a line parallel to the x axis passing through the midpoint of the trailing edge.

The geometric location of the center of pressure is shown in figure 5. Because the coefficient F cancels in the calculation of the center of pressure, the expressions obtained are exact.

The rolling moment about the bisector of the vertex angle (that is, $y_1 = 0$) may be obtained from equations (21) and (17). In terms of the maximum chord, the rolling-moment coefficient C_l is

$$C_l = \frac{\overline{y_1}}{x_c} C_L = - \frac{2\pi\alpha F}{3} \frac{\tan\theta \tan\psi}{\sqrt{1 - \tan^2\theta \tan^2\psi}} \qquad (22)$$

The pitching moment about the axis $\overline{x_1} = \frac{2}{3} x_c$ is zero for all values of yaw.

APPLICATIONS OF THEORY

In order to illustrate the effects of yaw, the pressure coefficient on a thin delta wing of vertex half-angle $\tan^{-1} 0.4$ at a Mach number $\sqrt{2}$ is presented in figure 6. In the computations, a corrected value of F (dotted curves of fig. 4) was used. The pressure coefficient seems to remain nearly constant along the center line and approaches infinity near either leading edge. As the angle of yaw is increased, the pressures on the least-swept side of the wing increase whereas the pressures on the most-swept side decrease. For the case $\psi = \theta$, the pressure coefficient on the most-swept leading edge (90° sweepback) becomes 0. (See equation (16).) For greater angles of yaw, the pressure coefficient is negative over a portion of the wing surface. (The solution obtained when the sweepback of one of the leading edges is greater than 90° does not conform to the Kutta-Joukowski condition for so-called subsonic trailing edges. For this reason, the $\psi = 23.2°$ curve is dashed.)

Corresponding to the shift in the pressure distribution (and center of pressure), a change in the lift and drag coefficients occurs. (See equations (17), (18), and (19).) The variation of the lift-curve slopes $\partial C_L/\partial\alpha$ and $\partial C_L/\partial\alpha'$ are presented in figure 7. In terms of the flow-deflection angle of attack α, the lift-curve slope increases with angle of yaw. In terms of the geometric angle of attack α' (the wing considered to be rotated about its base), the lift-curve slope decreases with angle of yaw.

The rolling-moment coefficients of the wing about the bisector of the vertex half-angle are presented as a function of angle of yaw in figure 8. These coefficients were calculated from equations (20) and (22) for $M = \sqrt{2}$ and $\theta = \tan^{-1} 0.4$. The rolling-moment coefficient is seen to be proportional to the angle of attack and is nearly proportional to the angle of yaw. A considerable rolling moment develops at large angles of yaw.

RESULTS OF ANALYSIS

An analysis of the yawed delta wing at supersonic speeds confined within the Mach cone from the vertex gave the following results:

1. Yawing a delta wing shifts the pressure distribution in such a manner as to increase the magnitude of the pressure coefficient on the least-swept side of the wing and decrease the pressure coefficient on the most-swept side of the wing.

2. The center of pressure of the yawed delta wing lies along a line parallel to the free-stream direction passing through the center of the delta base and at a distance two-thirds of the maximum chord from the vertex. A considerable rolling moment may therefore be experienced at large angles of yaw.

3. The lift-curve slope of the delta wing generally depends on the Mach number, the angle of sweepback, and the angle of yaw. The lift-curve slope of a delta wing of semi-vertex angle $\tan^{-1} 0.4$ at a Mach number of $\sqrt{2}$ either increases or decreases with angle of yaw, depending upon whether the flow-deflection angle of attack or the geometric angle of attack is used. The ratio of the two lift-curve slopes is the cosine of the yaw angle.

4. The absolute value of the ratio of lift coefficient to pressure coefficient at the center of the yawed thin flat-plate delta wing is found to be π.

Flight Propulsion Research Laboratory,
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APPENDIX - CALCULATION OF AERODYNAMIC COEFFICIENTS OF

YAWED DELTA WING

A set of Cartesian coordinates (x_1, y_1) defined on the wing surface is convenient for calculating the lift coefficient of the thin delta wing. The transformation equations are

$$
\left. \begin{array}{l}
x = x_1 \cos\psi + y_1 \sin\psi \\[2ex]
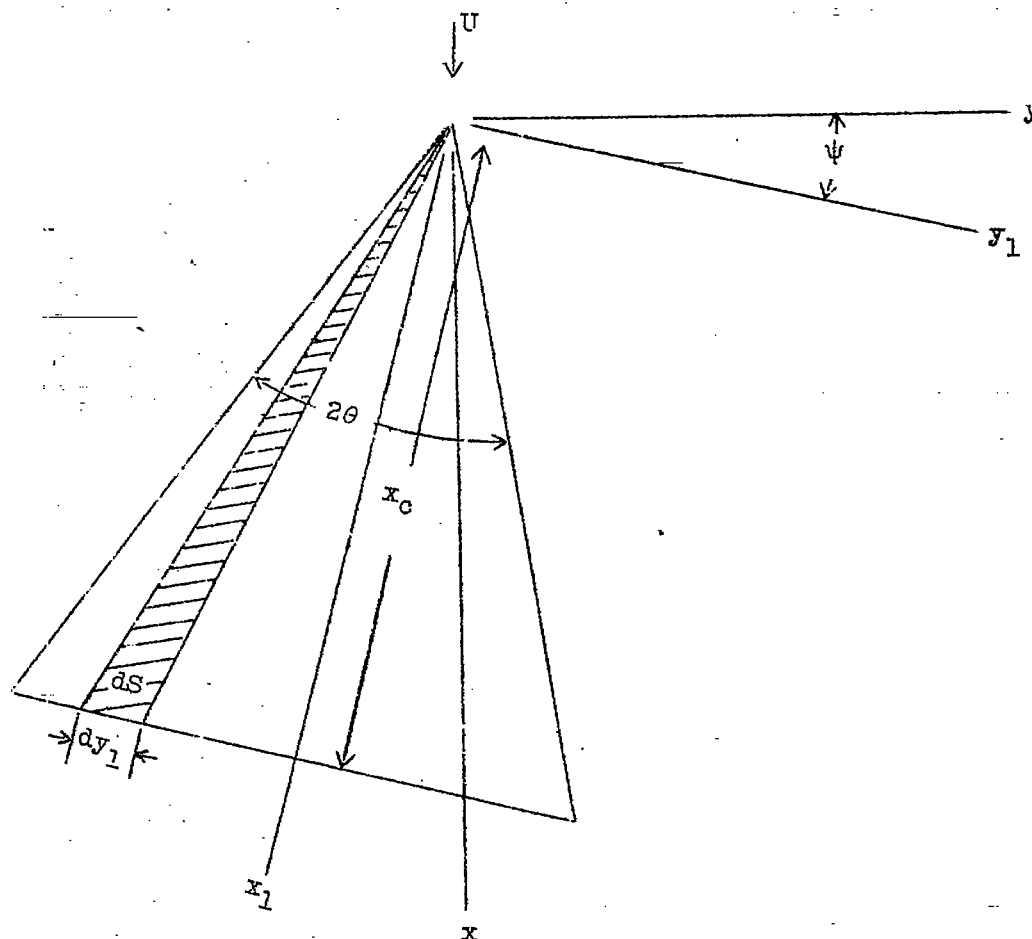y = y_1 \cos\psi - x_1 \sin\psi
\end{array} \right\} \tag{A1}
$$

The quantity y/x then becomes

$$
\frac{y}{x} = \frac{\dfrac{y_1}{x_1} - \tan\psi}{\dfrac{y_1}{x_1}\tan\psi + 1} \tag{A2}
$$

The pressure coefficient, equation (13), then becomes

$$
C_p = -\alpha F \sqrt{\frac{\tan^2\theta - \tan^2\psi}{1 - \tan^2\theta \, \tan^2\psi}} \left[\sqrt{\frac{\left(\dfrac{\tan\theta}{\tan\theta+\tan\psi}\right) + \left(\dfrac{1}{\tan\theta+\tan\psi}\right)\dfrac{y_1}{x_1}}{\left(\dfrac{\tan\theta}{\tan\theta-\tan\psi}\right) - \left(\dfrac{1}{\tan\theta-\tan\psi}\right)\dfrac{y_1}{x_1}}} \right.
$$

$$
\left. + \sqrt{\frac{\left(\dfrac{\tan\theta}{\tan\theta-\tan\psi}\right) - \left(\dfrac{1}{\tan\theta-\tan\psi}\right)\dfrac{y_1}{x_1}}{\left(\dfrac{\tan\theta}{\tan\theta+\tan\psi}\right) + \left(\dfrac{1}{\tan\theta+\tan\psi}\right)\dfrac{y_1}{x_1}}} \right] \tag{A3}
$$

Because the flow is conical, the lift may be evaluated by using a triangular infinitesimal area $dS = \dfrac{x_c \, dy_1}{2}$, where x_c is the maximum chord. The coordinates and the infinitesimal area are shown in the following sketch:

The lift coefficient is given as

$$C_L = \frac{\displaystyle\int 2C_p \, dS}{\displaystyle\int dS} = \frac{\displaystyle\int_{-x_c \tan\theta}^{x_c \tan\theta} -C_p \, x_c dy_1}{x_c^2 \tan\theta} = -\frac{1}{\tan\theta} \int_{-\tan\theta}^{\tan\theta} C_p \, d\!\left(\frac{y_1}{x_c}\right) \quad (A4)$$

Substitution of C_p from equation (A3) into (A4) and integration by formulas 111 and 113 of reference 6 yields

$$C_L = \frac{2\pi\alpha F \tan\theta}{\sqrt{1-\tan^2\theta \, \tan^2\psi}} \tag{A5}$$

provided that

$$|\theta \pm \psi| \le \tan^{-1} \frac{1}{\beta}$$

The center of pressure $(\overline{x_1}, \overline{y_1})$ will lie along the mean chord or $\overline{x_1} = \frac{2}{3} x_c$ from the vertex. The other coordinate of the center of pressure may be obtained from an evaluation of the expression.

$$\frac{\overline{y_1}}{x_1} = \frac{\displaystyle\int_{-\tan\theta}^{\tan\theta} C_p\left(\frac{y_1}{x_1}\right) d\left(\frac{y_1}{x_1}\right)}{\displaystyle\int_{-\tan\theta}^{\tan\theta} C_p \, d\left(\frac{y_1}{x_1}\right)} \tag{A6}$$

The integral in the numerator may be evaluated by equations 112, 113, and 172 of reference 6 to give (for the case $\psi < \theta$)

$$\int_{-\tan\theta}^{\tan\theta} \left[\sqrt{\frac{\left(\frac{\tan\theta}{\tan\theta-\tan\psi}\right) + \left(\frac{1}{\tan\theta+\tan\psi}\right)\frac{y_1}{x_1}}{\left(\frac{\tan\theta}{\tan\theta-\tan\psi}\right) - \left(\frac{1}{\tan\theta-\tan\psi}\right)\frac{y_1}{x_1}}} \right.$$

$$\left. + \sqrt{\frac{\left(\frac{\tan\theta}{\tan\theta-\tan\psi}\right) - \left(\frac{1}{\tan\theta-\tan\psi}\right)\frac{y_1}{x_1}}{\left(\frac{\tan\theta}{\tan\theta+\tan\psi}\right) + \left(\frac{1}{\tan\theta+\tan\psi}\right)\frac{y_1}{x_1}}} \right] \left(\frac{y_1}{x_1}\right) d\left(\frac{y_1}{x_1}\right) = -\frac{\pi \tan\psi \tan^2\theta}{\sqrt{\tan^2\theta - \tan^2\psi}}$$

$$\tag{A7}$$

The integral of the denominator may be evaluated by formulas 111 and 113 of reference 6 to give

$$\int_{-\tan\theta}^{\tan\theta}\left[\sqrt{\frac{\left(\frac{\tan\theta}{\tan\theta+\tan\psi}\right)+\left(\frac{1}{\tan\theta+\tan\psi}\right)\frac{y_1}{x_1}}{\left(\frac{\tan\theta}{\tan\theta-\tan\psi}\right)-\left(\frac{1}{\tan\theta-\tan\psi}\right)\frac{y_1}{x_1}}}\right.$$

$$\left.+\sqrt{\frac{\left(\frac{\tan\theta}{\tan\theta-\tan\psi}\right)-\left(\frac{1}{\tan\theta-\tan\psi}\right)\frac{y_1}{x_1}}{\left(\frac{\tan\theta}{\tan\theta+\tan\psi}\right)+\left(\frac{1}{\tan\theta+\tan\psi}\right)\frac{y_1}{x_1}}}\right]d\left(\frac{y_1}{x_1}\right)=\frac{2\pi\tan^2\theta}{\sqrt{\tan^2\theta-\tan^2\psi}}$$

$$\text{(A8)}$$

Therefore,

$$\frac{\overline{y_1}}{\overline{x_1}} = -\frac{\tan\psi}{2} \tag{A9}$$

(A similar derivation, with due regard to the sign of the radical, shows that equation (A9) also applies when $\psi > \theta$ provided that the pressure coefficient may be represented by equation (A3).) The coordinates of the center of pressure are then

$$\left.\begin{array}{l}\overline{x_1} = \dfrac{2}{3}\, x_c \\[3mm] \overline{y_1} = -\dfrac{x_c}{3}\tan\psi\end{array}\right\} \tag{A10}$$

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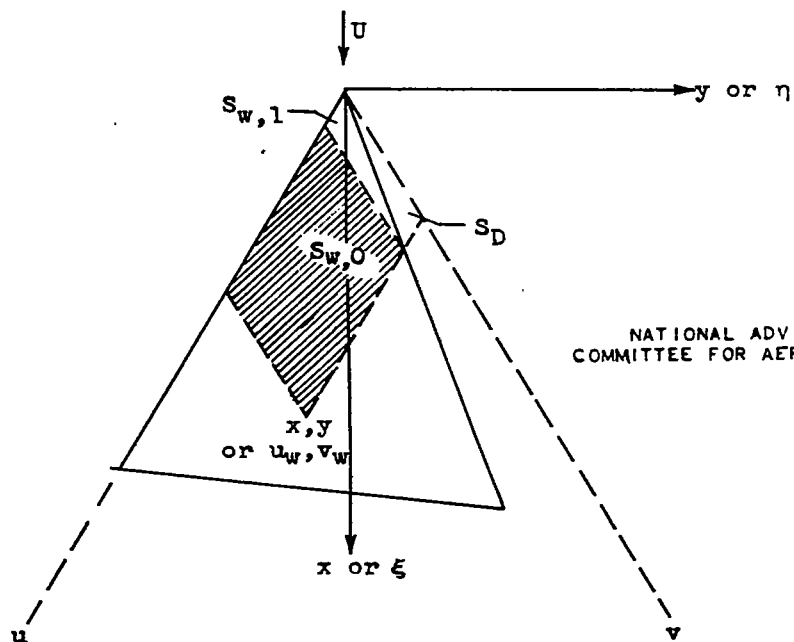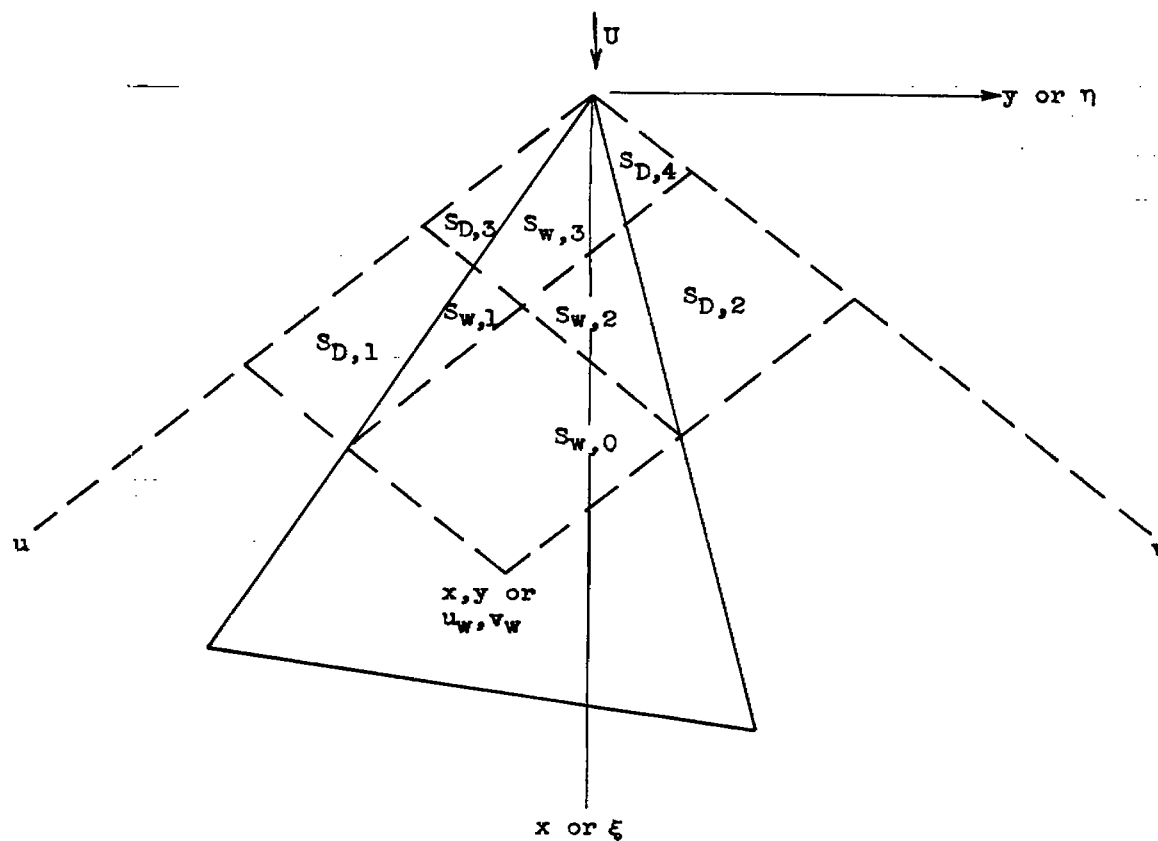   3rd ed. 1929, p. 18.
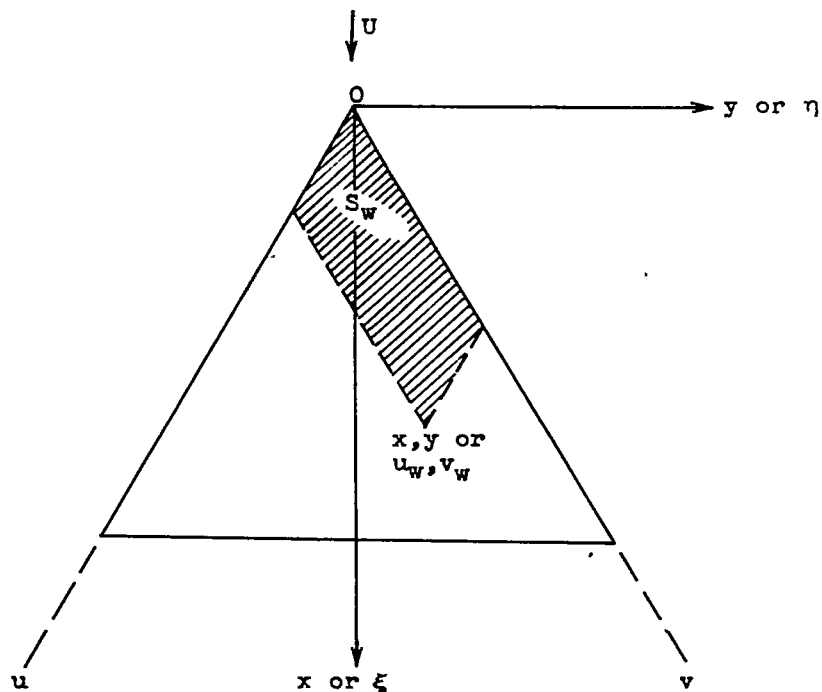
(a) Wing swept back along Mach lines without yaw.

(b) Wing so yawed that one leading edge coincides with
    Mach line $v = 0$ and with other leading edge swept
    behind Mach line $u = 0$.

Figure I. - Parallelogram representation of delta wing.

Fig. Ic

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(c) Yawed wing with both leading edges swept behind Mach line.

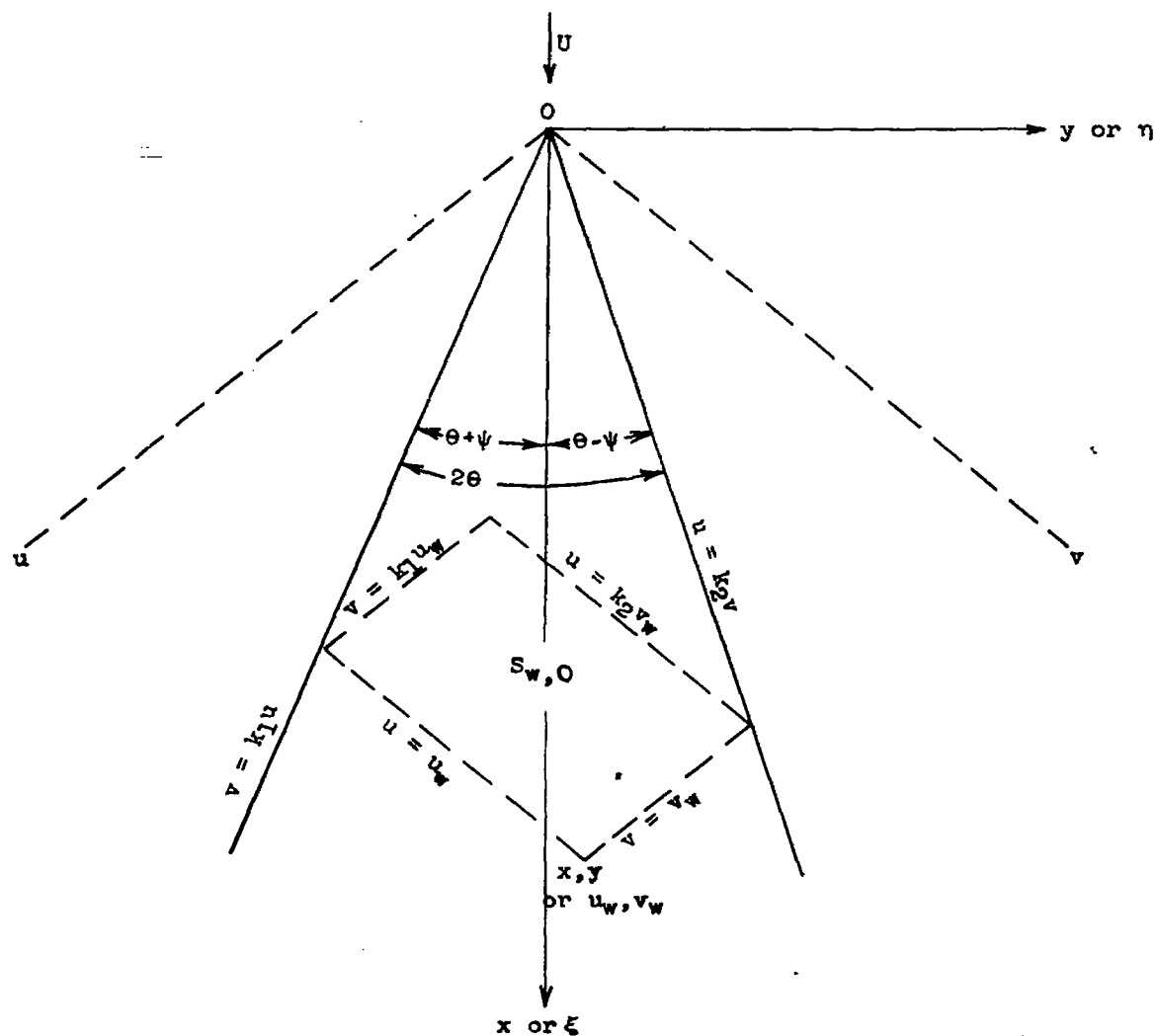Figure 1. - Concluded.   Parallelogram representation of delta wing.
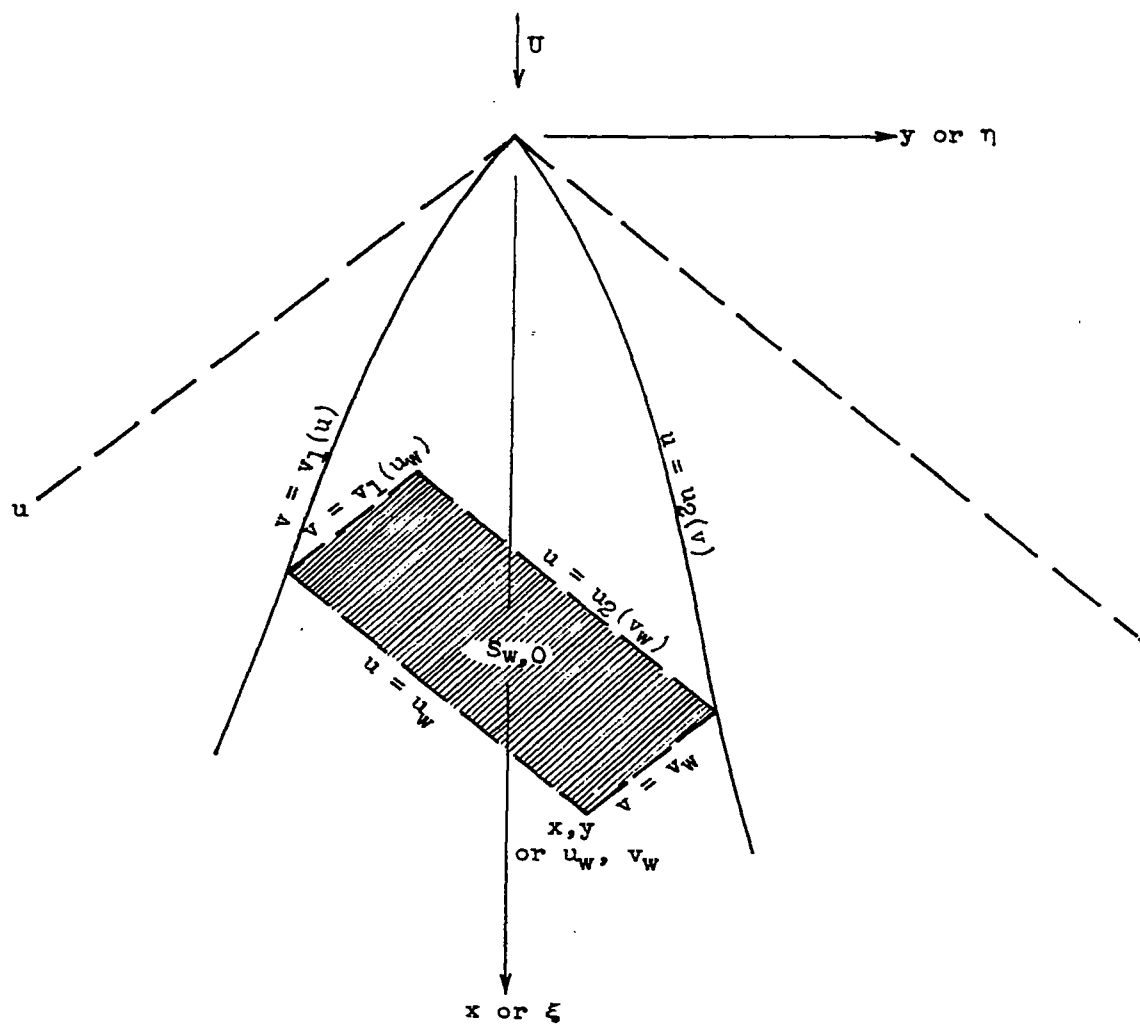
NATIONAL ADVISORY
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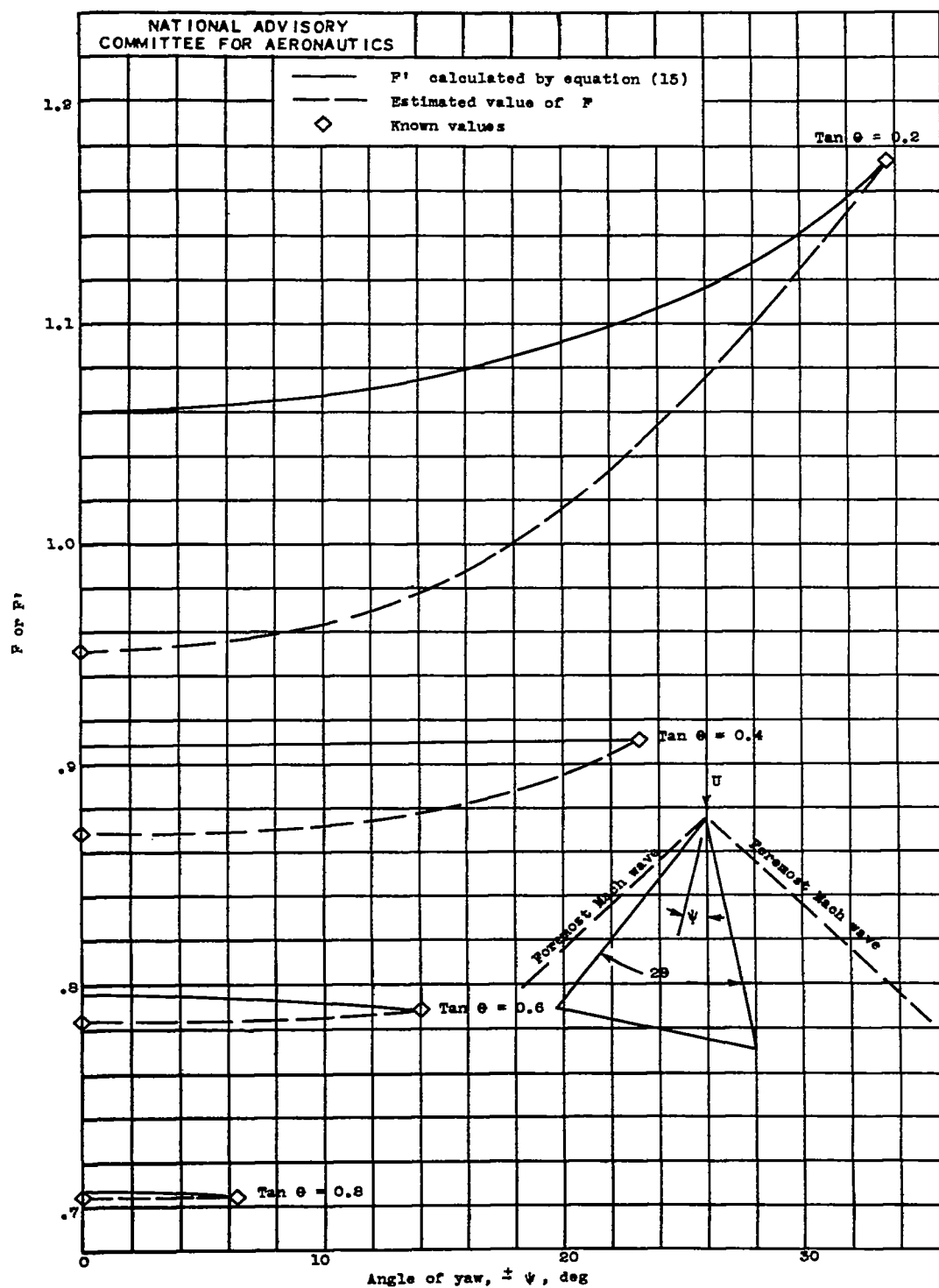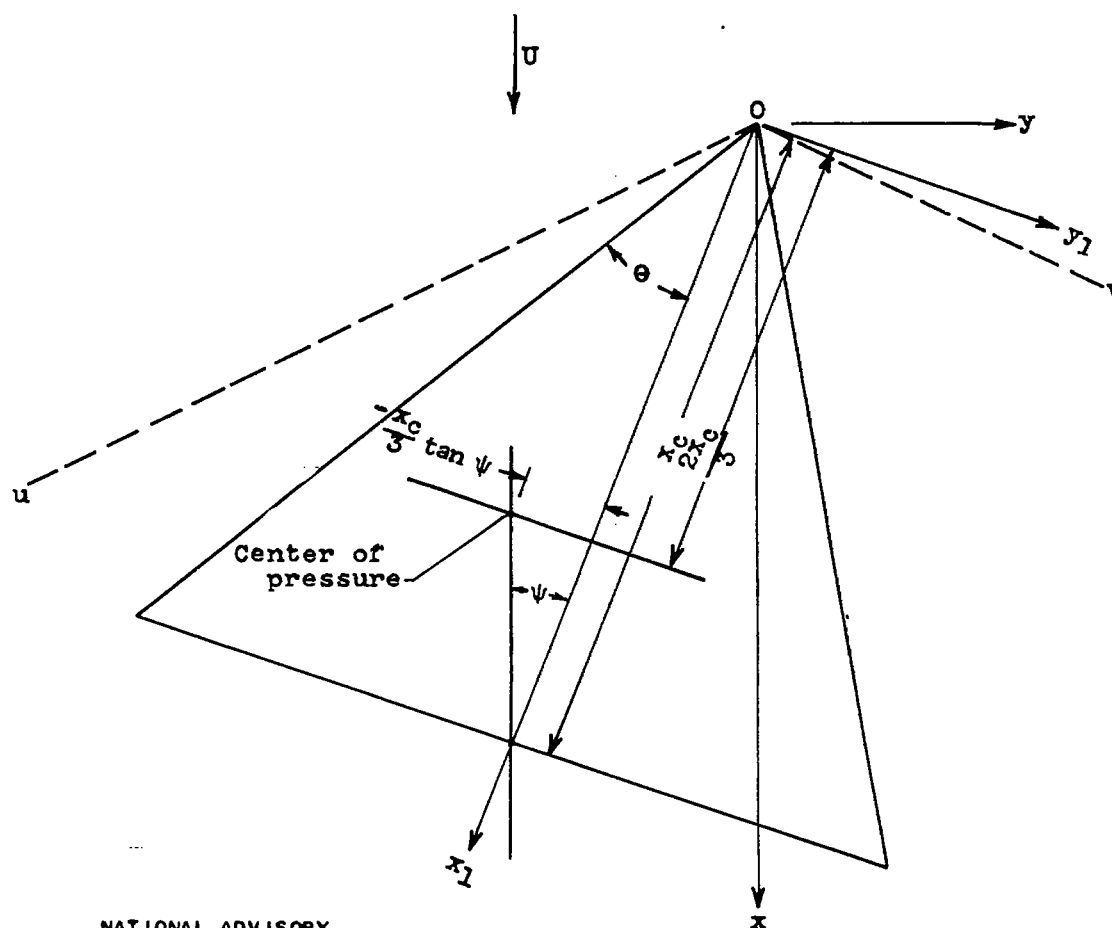Figure 2. – Field of integration for parallelogram representation of the pointed wing with curved leading edges.

Fig. 3

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Figure 3. – Field of integration for parallelogram representation of delta wing.

Figure 4. - Variation of the factor F with angle of yaw ψ at Mach number = √2.

Fig. 5   NACA TN No. 1429

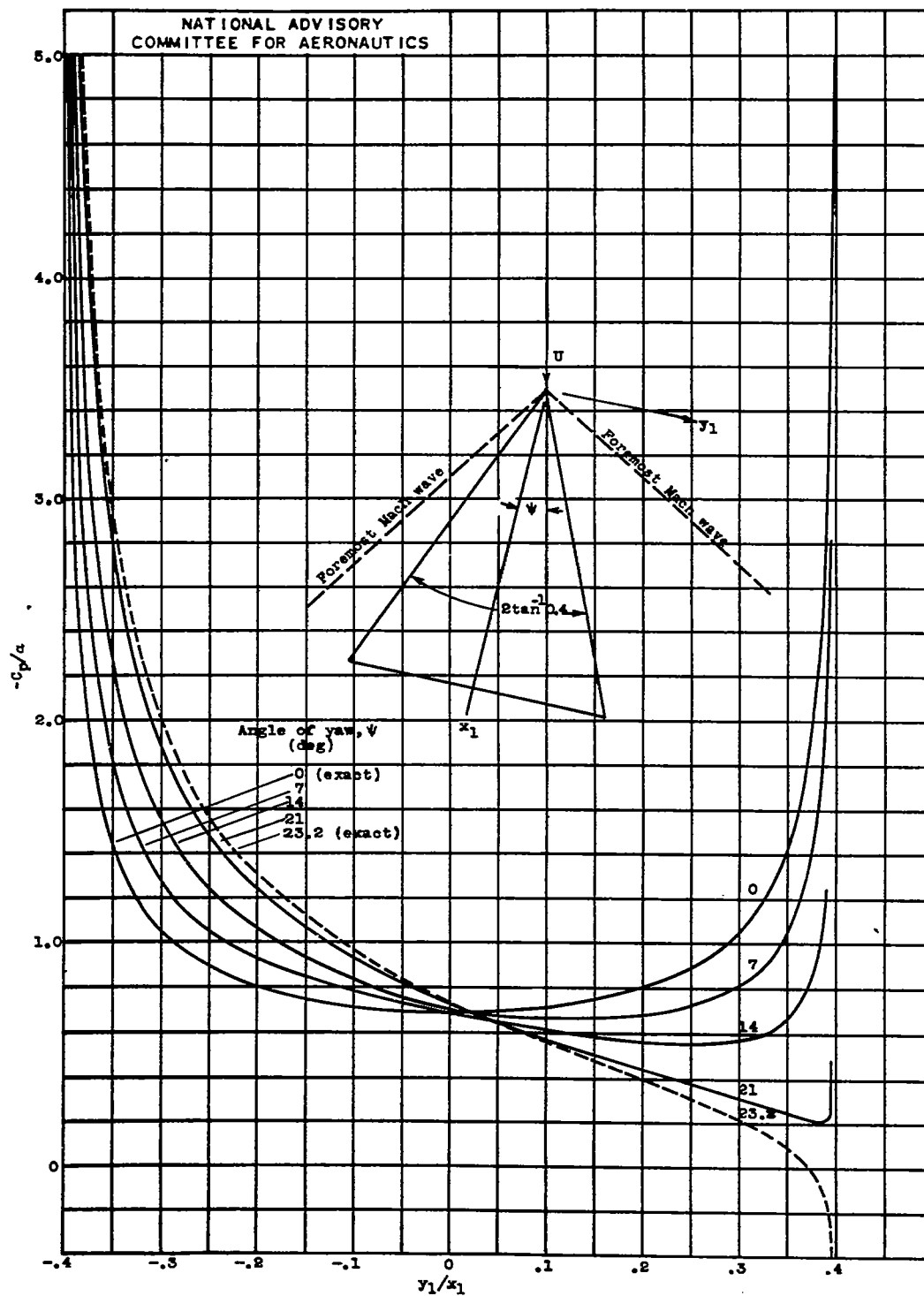Figure 5. - Location of center of pressure on yawed delta wing.

Figure 6. - Pressure distribution on delta wing at several angles of yaw
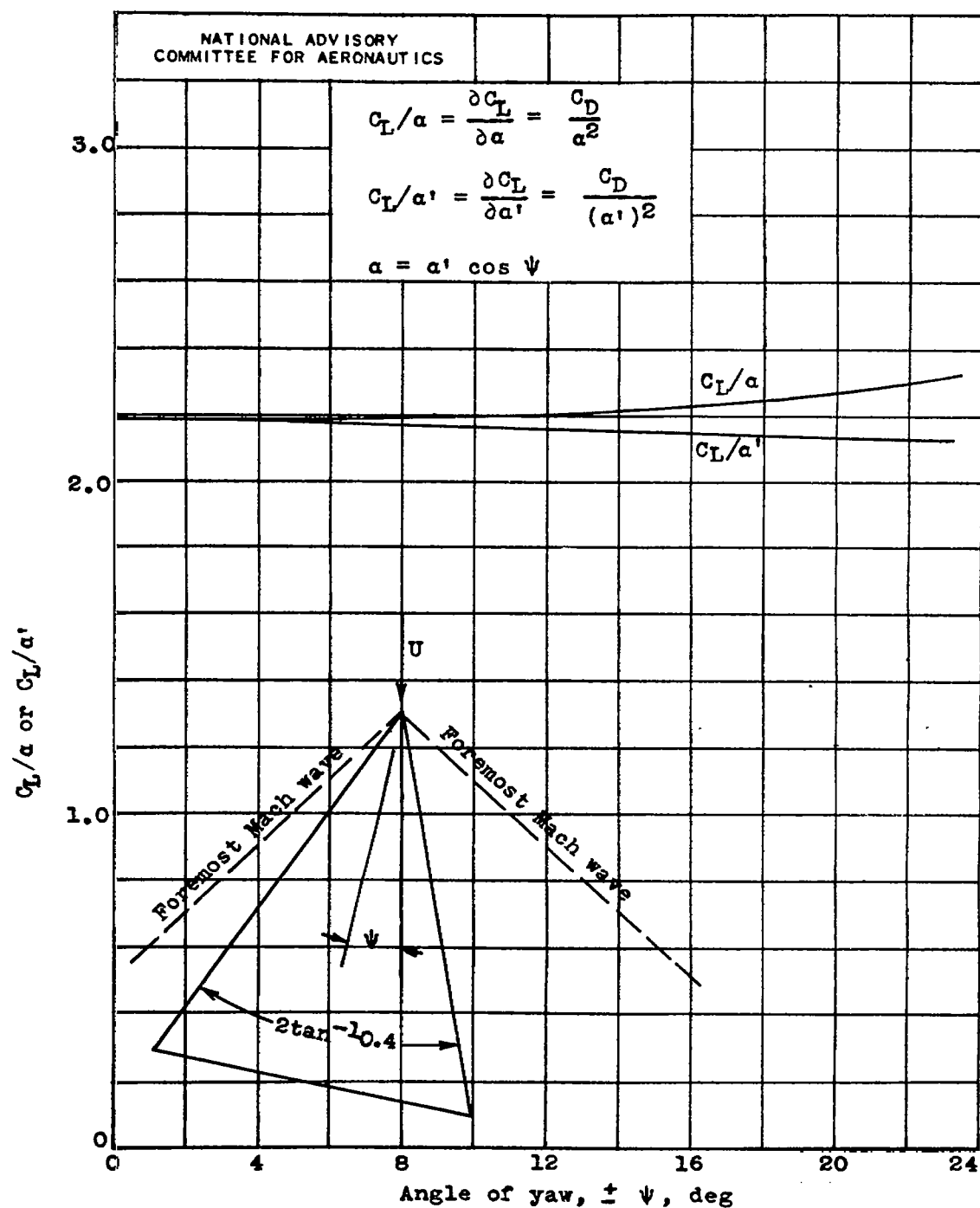for Mach number $\sqrt{2}$. Angle of attack α measured in radians.

Fig. 7   NACA TN No. 1429

Figure 7. – Variation of lift coefficient C_L and wave drag coefficient C_D with angle of yaw ψ for Mach number $\sqrt{2}$. Angle of attack α measured in radians.
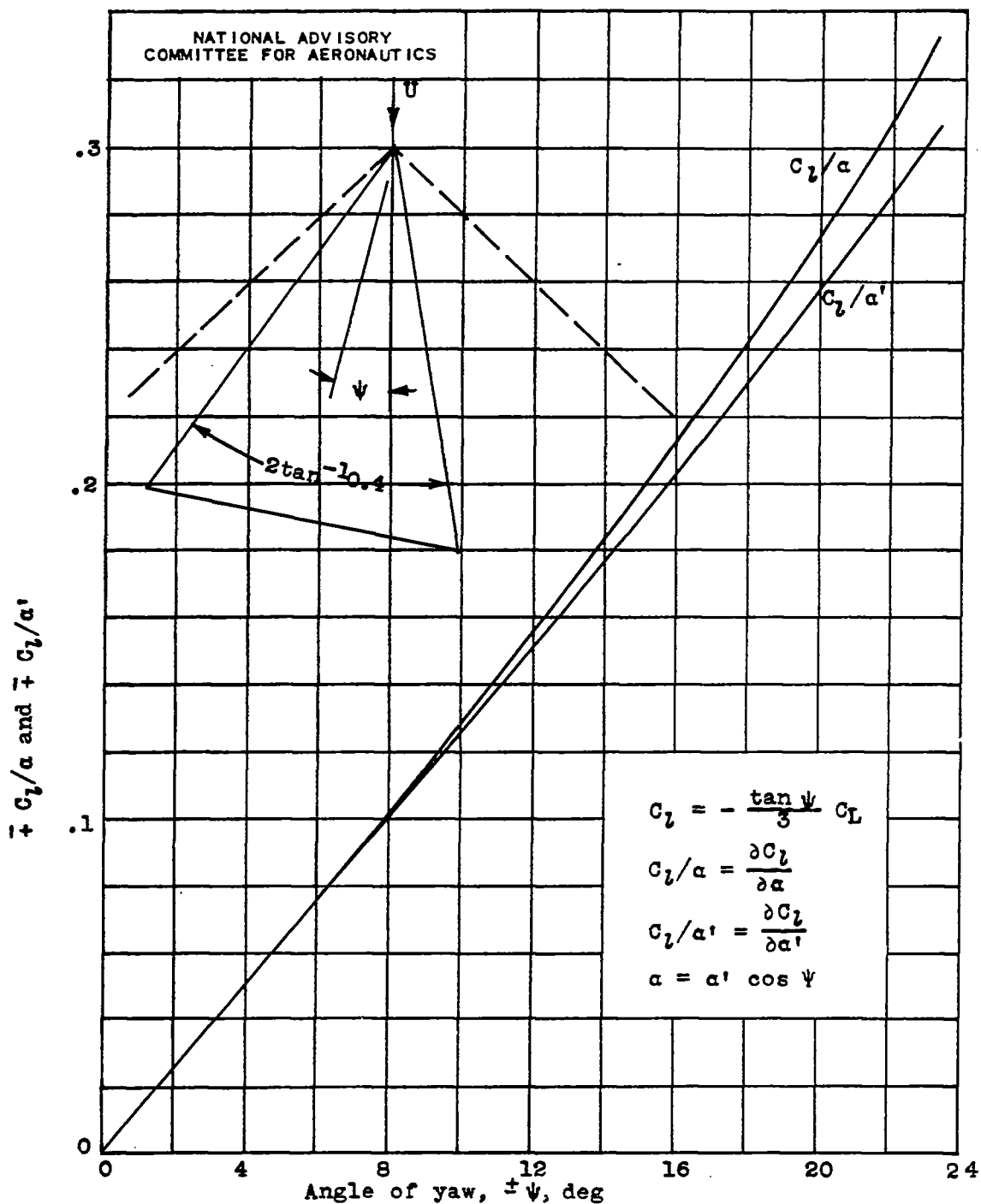
Figure 8. – Variation of rolling moment C_l with angle of yaw ψ for Mach number $\sqrt{2}$. Angle of attack α measured in radians.